$\mathcal{L}^k$ Zero-One Law

Logic and random structures have been widely studied in the literature. The first result that inspired researchers to do research in this area was indepedenly by Fagin [1] in 1976 and by Glebskii, Kogan, Liogon'kiI and Talanov [2] in 1969.

The Erdős-Rényi model of random graphs $G(n, p)$ is basically a graph obtained by taking the empty graph on n vertices and adding edges between any two vertices independently with probability p. Let FO be the first-order logic and let $\mathcal{L}$ be the set of all formulas of FO. Given a formula of $\varphi \in \mathcal{L}$, the event "random graph $G(n, p)$ satisfies φ" is denoted by $G(n, p) \models \varphi$. Given p = p(n), the $\mathcal{L}$ *zero-one law* is said to hold on $G(n, p)$ if the limiting probability $\mathsf{P}(G(n, p) \models \varphi) \longrightarrow a \in \{0, 1\}$, for any $\varphi \in \mathcal{L}$. The group of author cited above proved that for a constant probability p, the $\mathcal{L}$ zero-one law holds.

In 1988, Spencer and Shelah [3] proved that for $p = n^{-\alpha}$, where $\alpha \in (0, 1) \cap \mathbb{R} \backslash \mathbb{Q}$, the $\mathcal{L}$ zero-one law holds.

We denote by $\mathcal{L}^k$ the portion of the formulas of FO having at most k variables. My main objective for this project is to investigate whether the $\mathcal{L}^k$ zero-one or the $\mathcal{L}^k$ convergence law (the limiting probability is a constant $c \in (0, 1)$) are satisfied and to find for which minimal $k \in \{3, 4\}$, there exists a formula $\phi \in \mathcal{L}^k$ such that the set

$$\mathrm{S}^1(\phi) = \{\alpha \in (0, 1) \mid \mathsf{P}(G(n, p) \models \phi) \not\rightarrow c \in \{0, 1\}\}$$

is infinite. The set $\mathrm{S}^1(\phi)$ is the *spectrum* of ϕ.

References

[1] Ronald Fagin. Probabilities on finite models 1. *The Journal of Symbolic Logic*, 41(1):50–58, 1976.

[2] Yu V Glebskii, Do I Kogan, MI Liogon'kiI, and VA Talanov. Range and degree of realizability of formulas in the restricted predicate calculus. *Cybernetics*, 5(2):142–154, 1969.

[3] Saharon Shelah and Joel Spencer. Zero-one laws for sparse random graphs. *Journal of the American Mathematical Society*, 1(1):97–115, 1988.